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1969 J. Phys. A: Gen. Phys. 2 330

(http://iopscience.iop.org/0022-3689/2/3/012)

Heating of electrons in a weakly ionized plasma by high-frequency electromagnetic waves

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MS. received 20*th May* 1968, *in revised form* 30*th October* 1968

Abstract. The plasma perturbation in the presence of a high-frequency electromagnetic field is represented by a Maxwellian distribution function. The electron energy gained in the plasma in the presence of the applied electromagnetic field is calculated from the Boltzmann transport equation. The variation of electron energy with the duration of the field is calculated. It is shown that the electromagnetic field interaction with an extended plasma results in a steady-state electron temperature. The time for the electron temperature to reach the steady state is found to depend on the electron collision frequency.

Pinskii (1964) calculated the heating of electrons due to the interaction of electromagnetic waves with a plasma. Sturrock (1966) and Puri (1966) have used different techniques to calculate the heating of electrons produced by stochastic electromagnetic fields. Pinskii in his paper has not mentioned what collision term he has used for the computation of electron heating. Therefore the present authors decided to calculate the heating of electrons and to study how the rate of heating changes with the collisional term adopted. In the presence of electromagnetic waves electrons gain energy and lose energy to ambient particles in every collision. If the energy of the moving electrons is $\frac{1}{2}mu^2$, the rate of change of the electron energy can be written as

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = \int \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}} \frac{mu^2}{2}\, \mathrm{d}^3u\, \mathrm{d}^3x. \tag{1}$$

The rate of electron energy loss can be computed only when we know the nature of the electron velocity distribution function $f(r, v, t)$ in the presence of electromagnetic waves and the collisional term characteristic of perturbation which arises due to the applied field. We assume the electron velocity distribution function to be of the form

$$f(r, v, t) = \left(\frac{m}{2\pi kT}\right)^{3/2} N \frac{\exp\{-(mu^2+2P)/2kT\}}{\int \exp(-P/kT)\, \mathrm{d}^3x} \tag{2}$$

where N is the total number of electrons, T is the electron temperature and $P = ma^2/2\omega^2$; a is the acceleration experienced by electrons in the presence of applied electromagnetic waves. The collisional term used in the present calculation is (Desloge and Matthysse 1960)

$$\left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}} = -\nu(f-f_0) + \frac{m}{Mv^2}\frac{\partial}{\partial v}(\nu v^2 f_0) + \frac{kT}{mv^2}\frac{\partial}{\partial v}\left(\nu v^2 \frac{\partial f_0}{\partial t}\right) \tag{3}$$

where ν is the frequency of collision of electrons with neutral particles, M is the mass of the molecules, k is the Boltzmann constant and T is the absolute temperature in °K. This collisional term consists of four different terms. These were substituted into equation (1), and after integration we obtain

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = \frac{3}{8\pi}\nu NkT \frac{m}{M}\left\{\frac{M}{4\omega^2 kT}\frac{\int a^2 \exp(-P/kT)\, \mathrm{d}^3x}{\int \exp(-P/kT)\, \mathrm{d}^3x} - 1\right\}. \tag{4}$$

This expression is very similar to the one obtained by Pinskii except for an additional factor $1/8\pi$ appearing on the right-hand side. The first term inside the brace on the right-hand side also differs by a factor of $\frac{2}{3}$. This difference arises owing to different values of $(\partial f/\partial t)_{\text{coll}}$, which is characteristic of the applied fields and other plasma parameters. Pinskii, however, has not mentioned what value he used for $(\partial f/\partial t)_{\text{coll}}$. Our calculations show that the times needed for the plasma to reach the steady state and the final temperature of electrons in the plasma are strongly dependent on the assumption of the velocity distribution function and the collisional terms. In calculating the energy variation with time the electron collisional frequencies are assumed to be space and velocity independent. From equation (4) the steady-state temperature is obtained by setting $\mathrm{d}\epsilon/\mathrm{d}t = 0$ and thus we have

$$\frac{M}{4\omega^2 kT} \frac{\int a^2 \exp(-P/kT)\, \mathrm{d}^3x}{\int \exp(-P/kT)\, \mathrm{d}^3x} = 1. \tag{5}$$

The acceleration experienced by electrons in the presence of a high-frequency electromagnetic field is expressed in terms of spatial perturbation. Considering the electromagnetic field to be in the form of short duration pulses and following the technique of Pinskii, we have evaluated the integrals in equation (4). The resulting expression for the steady-state electron temperature, with similar assumptions, is

$$T = \frac{M a_0{}^2}{4\omega^2 k\{1-(M/m\gamma)(1+\beta)\}} \tag{6}$$

where γ and β are constants as used by Pinskii. It is evident that the steady state is possible only when

$$\gamma > \frac{M}{m}(1+\beta).$$

The variation of the steady-state electron temperature with the magnitude of the electromagnetic field is computed for a fixed frequency and various values of the steepness parameter γ. The electron temperature is found to increase with increasing field strength of the electromagnetic wave, as shown in figure 1. For low values of γ the electromagnetic

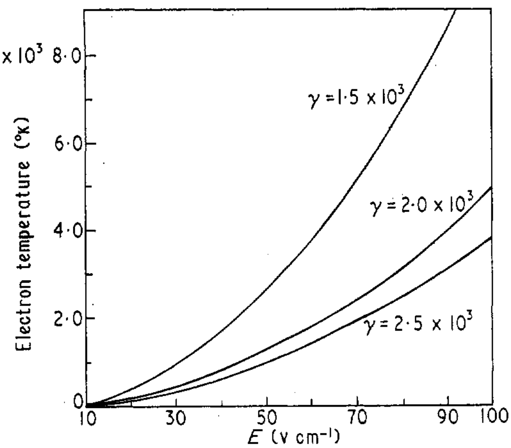

Figure 1. Variation of the steady-state electron temperature with the strength of electromagnetic fields for $f = 20\,000$ MHz and $\beta = 0$.

field pulses are less steep and the electron temperature increases rapidly and attains a higher temperature, whereas for steeper electromagnetic field pulses the electron temperature increases comparatively slowly and attains a lower temperature. The variation in the electron temperature with the frequency of the electromagnetic field is computed for a fixed value of field strength and is shown in figure 2 for different values of the steepness parameter. The electron temperature is found to decrease with increasing frequency of the electromagnetic field. The electron temperature decreases rapidly from a high value with increasing frequency of the electromagnetic field, and finally the electron temperature attains a steady value for electromagnetic fields of very high frequency. The decrease in

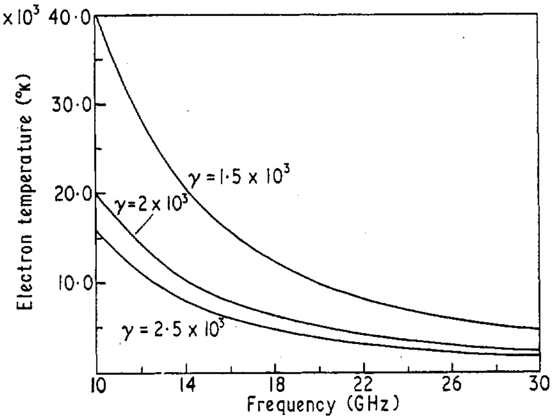

Figure 2. Variation of the steady-state electron temperature with the frequency of electromagnetic fields for $E = 100 \text{ v cm}^{-1}$ and $\beta = 0$.

electron temperature is less rapid for higher values of the steepness parameter. The rate of energy loss can further be written as

$$\frac{d\epsilon}{dt} = \frac{3}{32\pi} \frac{\nu N m a_0^2}{\omega^2} + \frac{3}{8\pi}\left(\frac{1}{\gamma} - \frac{m}{M}\right) \nu N k T. \tag{7}$$

Integrating equation (6) and considering the initial plasma temperature to be T_0, we solve the resulting expression for time:

$$t = \frac{1}{(3/8\pi)\nu(1/\gamma - m/M)} \ln\left\{\frac{ma_0^2/4\omega^2 + (1/\gamma - m/M)kT}{ma_0^2/4\omega^2 + (1/\gamma - m/M)kT_0}\right\}. \tag{8}$$

Similar integration of Pinskii's energy equation gives

$$t' = \frac{1}{\nu(2/\gamma - 3m/M)} \ln\left\{\frac{ma_0^2/2\omega^2 + (2/\gamma - 3m/M)kT}{ma_0^2/2\omega^2 + (2/\gamma - 3m/M)kT_0}\right\}. \tag{9}$$

The change in electron energy with time is calculated from equations (8) and (9), on the assumption that $T_0 = 500$ °K. It is shown by two sets of curves in figure 3. Our calculation

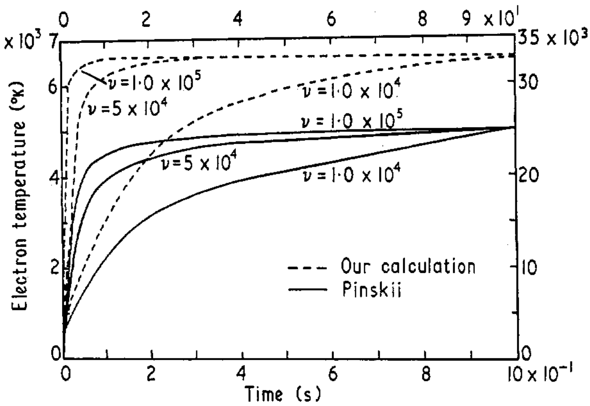

Figure 3. Variation of electron temperature with time for $f = 20\,000$ MHz, $E = 100 \text{ v cm}^{-1}$, $\gamma = 2{\cdot}5 \times 10^3$ and $\beta = 0$.

shows that the time required for the plasma to reach the steady-state temperature is approximately one hundred times more than the time obtained from the computations following Pinskii's work (equation (9) in this paper). Therefore it can be concluded that the fully ionized plasma can attain higher temperature compared with a weakly ionized plasma, provided the strength and the duration of the electromagnetic pulse is the same. However, the increase in electron temperature with time is very rapid for high collision frequency. The application of such a high field in the laboratory may be handicapped by a plasma sheath formation around the input probe. This may be more realistic for an astrophysical plasma being heated by naturally generated electromagnetic waves. Recent spacecraft experiments have detected high-energy electrons in the transition region between the magnetosphere and the bow shocks (Fan *et al.* 1964, Anderson *et al.* 1964). These high-energy electrons arise from the interaction of the oscillating electric field with the ambient plasma in the transition region (Bernstein *et al.* 1964, Scarf *et al.* 1965). The interactions of electromagnetic waves with the plasma produce heating of electrons in this region of the magnetosphere.

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